Probability

Students will:

- Determine the sample space for an event
- Construct and use tree diagrams to determine the number of outcomes or the probability of a given event
- Use the multiplication and addition principles to calculate the number of outcomes or the probability of events
- Find probability of simple events and of compound independent events
- Use technology and manipulatives to simulate probability events

Classroom Cases:

1. In the 110th Congress, there are 50 Democratic senators, 49 Republican senators, and one Independent senator. If five senators are selected at random to form a committee, determine the probabilities for the following possibilities:

a. the independent senator is on the committee
b. the committee consists of senators from one party
c. the committee consists of senators from more than one party

Case Closed - Evidence:

a. The probability of selecting the independent senator for one seat on the committee is 1/100. But there are five seats on the committee. So the probability of the independent senator winning one of those seats is 5 x 1/100 = 5/100 = 0.05

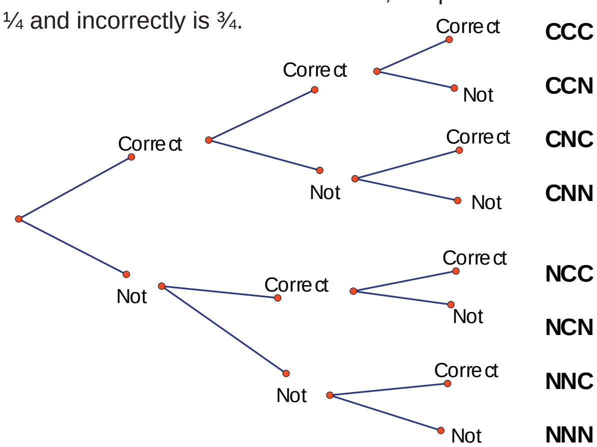
b. The committee could consist of all Democrats or all Republicans. We calculate P(5D) + P(5R) = (50/100 x 49/99 x 48/98 x 47/97 x 46/96) + (49/100 x 48/99 x 47/98 x 46/97 x 45/96) = 0.05437.

c. The committee could have any composition except all Democrats or all Republicans. This probability would therefore be the complement of the one computed in part b: 1 – 0.05437 = 0.94653

2. If you make random guesses for three multiple choice questions (each with four answers), what is the probability of getting exactly one correct? Of getting at least one correct? Support your answer by displaying the sample space.

Case Closed - Evidence:

Since only one of the four answer choices is correct, the probabilities of answering one question correctly is ¼ and incorrectly is ¾.



From the tree diagram and sample space above, I see that there are three ways to get exactly one correct answer. The probability for this is 3 x ¼ x ¾x ¾ = 27/64. From the sample space, I see that the probability of getting at least one correct is the same as the probability of not getting "none correct" or NNN. P(NNN) = ¾ x ¾ x¾ =27/64.
P(not getting NNN) = 1 – 27/64 = 37/64. This is the complement of P(none correct).

Further investigations:

Discuss with your student how probability influences decision-making. For example, how does the chance of rain affect your choice of clothing? How does the likelihood of favorable legislation influence your plans for increasing your farm or your business? How do market predictions impact your stock trading?

Explore with your student his chances of winning games such as SORRY!, Monopoly, YAHTZEE and backgammon. The web site http://media.pearsoncmg.com/aw/aw_triola_elemstats_9/ip/tes09_03_ip.htm can be helpful.

Ask your student to calculate the free throw percentage for her favorite basketball player. If that player is shooting a 2-point foul shot, what is the probability that the player makes both shots? What is the probability that he makes at least one of the shots?

Terminology:

Addition principle: To find the probability of one event occurring or of another independent event occurring, add their probabilities.

Independent events: Events for which the occurrence of one has no impact on the occurrence of the other.

Relative frequency: The number of times an outcome occurs divided by the total number of trials.

Sample space: All possible outcomes of a given experiment.

Event: A subset of a sample space.
Simple Event: An event consisting of just one outcome. A simple event can be represented by just one branch of a tree diagram.

Compound event: A sequence of simple events.

Complement: All outcomes that are possible for a given event, but are NOT desired. The complement of event E (noted E') occurs when E does not occur. P(E') =1 - P(E).

Counting principle: If event A can occur **m** ways and for each of these **m** ways, there are **n** ways event B can occur, then events A and B can occur ***m*** x ***n*** ways.

Multiplication principle: To find the probability that two independent events (A and B) occur, multiply their individual probabilities.

Tree diagram: A tree-shaped diagram that illustrates sequentially the possible outcomes of a given event.

Book 'em:

Do You Want to Bet? by Jean Cushman

Related Files:

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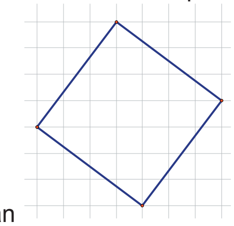
Exponents

Students will:

- Distinguish between rational and irrational numbers
- Estimate or find the square root of non-negative numbers including 0
- Locate square roots on a number line
- Use properties of real numbers and order of operations to simplify and evaluate simple numeric and algebraic expressions involving integer exponents
- Write large and small numbers using scientific notation
- Solve problems using the Pythagorean Theorem

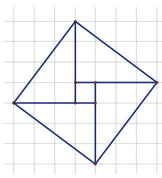
Classroom Cases:



1. Find the area of the square and the length of its side.

Case Closed - Evidence:

I can divide the square into four congruent triangles and a square with an area of 1 sq. unit as shown below. Then I can calculate the areas of the triangles using A = ½ x base x height.



The area of one triangle is ½ • 3 • 4 = 6 sq. units. The area of four triangles will be 4 • 6 = 24 sq. units.

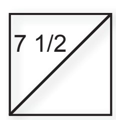
The total area of the square is 24 + 1 =25 sq. units.

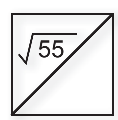
Since Area = side • side = side^2, 25 = side^2.

The length of a side of the square is √25 = 5 units.

2. Ms. Green is considering two square plots for flower beds. One has a diagonal of 7½ ft^2 and the other has a diagonal of √55 sq. ft. Which plot is larger? How much compost will the larger bed require? (Ms. Green wants 1 pound of compost for 3 sq. ft.) How much fencing will the larger bed need?

Case Closed - Evidence:





$$7\frac{1}{2}=\frac{15}{2} \qquad \left(\frac{15}{2}\right)^2=\frac{225}{4}=56\frac{1}{8}$$

Since $56\frac{1}{8} > 55$, the first plot is larger.

To determine how much compost Ms. Green will need, I first have to find the area of the plot. And to find the area I have to know the length of a side (a). Using the Pythagorean Theorem, hypotenuse^2 = side^2+side^2 or $c^2 = a^2 + a^2 = 2a^2$

Substituting: $c^2 = \left(\frac{15}{2}\right)^2 = \frac{225}{4} = 2a^2$

Dividing by 2: $\frac{225}{8} = a^2$

Taking sq. root: $\sqrt{\frac{225}{8}} = \frac{\sqrt{15\bullet 15}}{\sqrt{2\bullet 2\bullet 2}} = a$

Rationalizing: $\frac{15}{2\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{15\sqrt{2}}{4}$ feet $= a$

The area of the larger plot will be $a^2 = \frac{15\sqrt{2}}{4} \bullet \frac{15\sqrt{2}}{4} = \frac{15^2 \bullet \sqrt{4}}{4^2} = \frac{225\bullet 2}{16} = \frac{225}{8} = 28\frac{1}{8}$ sq. ft.

For the compost, Ms. Green will need $\frac{225}{8}$ sq. ft $\bullet \frac{1 \text{ pound}}{3 \text{ sq. ft.}} = \frac{225}{24} = 9\frac{3}{8}$ pounds.

For the fencing, Ms. Green needs the perimeter of the square: $P = 4a = 4 \bullet \frac{15\sqrt{2}}{4} = 15\sqrt{2} \approx 21.21$ feet.

3. The population of the world is about 6.15 x 10^9 people. How many communities with the same population as ours would it take to generate the world's population?

Case Closed - Evidence:

About 25000 people live here. 25000 = 2.5 x 10^4

$$\frac{6.15 \times 10^9}{2.5 \times 10^4} = 2.46 \times 10^5 = 246{,}000 \text{ communities}$$

Further investigations:

With your student, explore savings accounts. Make tables showing how much money will be in the accounts over time. Talk about how you could use exponents to figure out how much will be in an account at a given time.

Ask your student to use the Pythagorean Theorem to determine if the walls of a room in your home are "square" that is, two walls form a right angle.

When very large or very small numbers appear in the news, let your student write them in scientific notation. If the numbers are already in scientific notation, ask him to express the numbers in standard notation.

Terminology:

Additive inverse: The opposite of a number. 7 and -7 are additive inverses of each other.

Exponent: The number of times a base is used as a factor in repeated multiplication.

Hypotenuse: The side opposite the right angle in a right triangle.

Irrational: A real number that cannot be expressed as the ratio of two integers.

Leg: Either of the shorter sides of a right triangle.

Pythagorean theorem: A rule that relates the lengths of the sides of a right triangle.

Radical: The symbol √ used to indicate square roots. Also expressions that contain √ .

Rational: A real number that can be expressed as the ratio of two integers with a non-zero denominator.

Scientific notation: A way to represent real numbers as the product of a number between 1 and 10 and a power of 10.

Square roots: One of two equal factors of a non-negative number.

Book 'em

What's Your Angle, Pythagoras? by Julie Ellis

The Adventures of Penrose, the Mathematical Cat by Theoni Pappas

"President Garfield and the Pythagorean Theorem" in **The Mathematical Universe** by William Dunham

In the Next Three Seconds by Rowland Morgan and Rod Josey

The King's Chessboard by David Birch

One Grain of Rice by Demi

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GEORGIA DEPARTMENT OF EDUCATION
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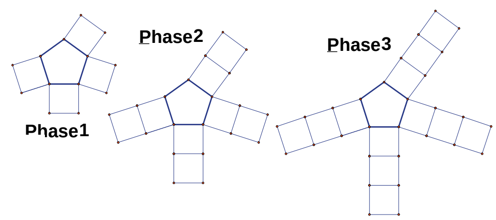
Linear Equations

Students will:

- Use algebraic expressions, equations, or inequalities in one variable to represent a given situation
- Simplify and evaluate algebraic expressions, including those with exponents
- Solve and interpret algebraic equations and inequalities in one variable, including those with absolute values
- Graph the solution of an equation or an inequality on a number line

Classroom Cases:

A new middle school is being built in phases. The first phase will have a core building with four classroom buildings attached as shown. As the population grows, more classroom buildings will be attached as shown in Phases 2 and 3. If the pattern continues, how many total buildings (core and classroom) would be needed in the seventh phase? In thenth phase?



Case Closed - Evidence:

Each additional phase increases the total number of buildings by 4.

Phase	Core + ? classrooms	Core + ? classrooms	No. of buildings
1	1+4	1+4(1)	5
2	1+4+4	1+4(2)	9
3	1+4+4+4	1+4(3)	13
4	1+4+4+4+4	1+4(4)	17
5	1+4+4+4+4+4	1+4(5)	21
6	1+4+4+4+4+4+4	1+4(6)	25
7	1+4+4+4+4+4+4+4	1+4(7)	29

There will be 29 buildings in the 7th phase. Following the pattern above, there will be 1+ 4***n*** buildings in thenth phase.

2. On his 13th birthday, Taylor had \$310 in his dresser drawer and decided to begin saving money to buy a used car. His uncle will sell him a car for \$2200. On the first day of each month, Taylor plans to add \$35 to his drawer. (Taylor's mother, an accountant, suggests that next time Taylor learn about interest-paying bank accounts.) How old will Taylor be when he can afford to buy his uncle's car? If Taylor waited until his 15th birthday to start saving for the car, how much would he have to save each month to buy the car at the same age?

Case Closed - Evidence:

Let n = number of months Taylor will have to save money for the car

Then $35n$ = amount saved each month

$310 + 35n = 2200$

$35n = 1890$

$n = 54$

Taylor will have to save for 54 months, or 4.5 years. He will be 13 + 4.5 = 17.5 years old when he can afford to buy his uncle's car.

If Taylor starts saving at age 15, he must save \$1890 in 17.5 – 15 = 2.5 years or 30 months.

Let a = amount saved each month

$30a = 1890$

$a = 63$

Starting at age15, Taylor would have to save \$63 each month.

3. In front of a new ride at the amusement park is a pole that is 160 cm tall. On the pole is a sign that says, "To ride this attraction your height must be within 30 cm of the height of this pole, inclusive."

Let ***h*** be the height of a rider and express the message on the sign algebraically using an absolute value inequality and using a compound inequality.

Case Closed - Evidence:

$|h – 160| < 30$ $h \geq 130$ and $h \leq 190$ which can be written $130 \leq h \leq 190$

4. Solve the following for ***x*** and graph the solution of the inequality on a number line:

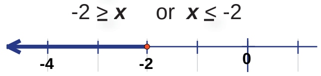
a. $7+x \leq 1 – 2x$ b. $8 – 3(x - 5) = 12$ c. $A = ½ h(x + b)$

Case Closed - Evidence:

a. $7+x \leq 1 – 2x$

$6 \leq -3x$

$-2 \geq x$ or $x \leq -2$



b. $8 – 3(x - 5) = 12$

$8 – 3x + 15 = 12$

$-3x + 22 = 12$

$-3x = -10$

$x = 10/3 = 3\ 1/3$

c. $A = ½ h(x + b)$

$2A = hx + hb$

$2A – hb = hx$

$\dfrac{2A – hb}{h} = x$

Further investigations:

If you use formulas in your work or daily activities, share them with your student and explain how and why you use them.

Show your student the formula for amount in a savings account ***A***=***P***(1+***rt***) where ***P*** is the principal deposited, ***r*** is the rate of interest, and t is the time in years. Supply different values for three of the variables, and ask your student to solve for the remaining variable.

Discuss with your student how you use inequalities. For example, I spend three times as much for groceries as I do for gasoline; I can spend at most \$150 for groceries and gasoline. How much can I spend on gasoline? Or, I spend between 65 and 120 minutes in my car each day; I make five trips a day. How long is a typical trip? Ask your student to represent such situations with symbols and on number lines and to solve for unknown quantities.

Terminology:

Absolute value: The distance a number is from zero on the number line. |-8| = 8 and |9|=9.

Addition property of equality: Adding the same number to both sides of an equation produces an equivalent equation.

Algebraic expression: A mathematical phrase that contains at least one variable.

Equation: A mathematical sentence that says that two mathematical expressions have the same value.

Evaluate an algebraic expression: Substitute values for the variables and simplify to obtain one value.

Inequality: A mathematical sentence using >,<, ≥, or ≤ to show that two mathematical expressions have different values.

Inverse operations: Pairs of mathematical operations that undo each other.

Like terms: Monomials that have the same variables raised to the same powers.

Multiplication property of equality: Multiplying both sides of an equation by the same non-zero number produces an equivalent equation.

Simplify an algebraic expression: Perform all operations possible.

Solve: Identify the values that when substituted for the variable make the equation or the inequality true.

Variable: A letter or symbol used to represent a number.

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Further investigations:

Have a contest to see who can identify the most arithmetic sequences. Consider license plates, bar codes, zip codes, telephone numbers, grocery receipts, meter readings. For each sequence, identify the common difference, graph the terms, and represent the sequence with recursive and explicit formulas.

List pairs of objects or relationships such as (husbands, wives), (customers, stores) or (mayors, cities). Do the pairs form functions?

Make up puzzles like Classroom Case 1. Draw Venn diagrams to solve the puzzles.

You have many sets around your home, such as sets of dishes, sets of silverware, swing sets, paint sets, sets of tools. Discuss with your student why these are called “sets'. What would the union of a set of dishes and a set of silverware include? How would you describe the intersection of a set of tools and a set of glasses? If you have art supplies, what would be in the complement of your paint set?

Terminology:

Complement of a set: The collection of all items not in the set

Element: A member or item in a set

Explicit form: An algebraic expression that produces terms of a sequence by substitution of the term numbers.

Function: A rule for matching elements of two sets in which an element from the first set matches only one element in the second set.

Intersection: The set of all elements contained in all the given sets.

Null set: A set that contains no elements. Also called an “empty set”.

Recursive form: A set of algebraic expressions that produce the next term in a sequence.

Relations: A rule that gives an output value for every valid input.

Set: A collection of numbers, geometric figures, letters, or other objects that have some common characteristic.

Subset: A collection of items drawn entirely from a single set. A subset can consist of any number of items from a set, ranging from none (null set) to the entire set.

Union: The set of all elements that belong to at least one of the given sets.

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Functions, Relations

Students will:

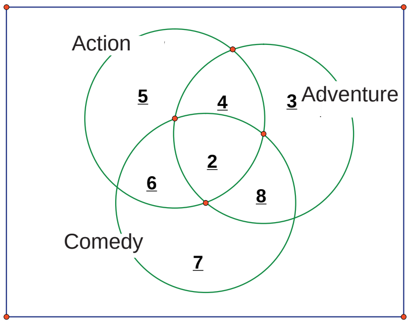
- Draw Venn diagrams with 2 or 3 circles and use them to compute set and subset membership and probabilities of membership
- Identify a correspondence between variables as a relation and determine if the relation is a function
- Translate among multiple representations of functions and relations
- Describe an arithmetic sequence as an example of a linear function and identify specific characteristics shared by arithmetic sequences and linear functions.

Classroom Cases:

1. A researcher interviewed a group of 55 students about the movies that they had seen recently. She determined the following:

17 had seen an “action” movie
17 had seen an “adventure” movie
23 had seen a “comedy”
2 had seen all of these movie genres
6 had seen an “action” movie and an “adventure” movie
8 had seen an “action” movie and a “comedy”
10 had seen an “adventure” movie and a “comedy”

How many students had seen exactly 2 of these movie genres? How many students had seen none of these movie genres? What is the probability that a randomly chosen student had seen exactly one of these movie genres?



Case Closed - Evidence:

In the intersections of each pair of circles, there are 4, 6, and 8 students for a total of 18 students who saw exactly 2 of the movies.

There are 35 students in the circles. 55-35=20. There are 20 students who had not seen any of these movies.

Out of the 55 students surveyed, 15 (5+3+7) had seen exactly one of these movies. The probability of randomly choosing a student who had seen exactly one of these movies is 15/55 = 3/11.

2. A car whose original value was $26500 decreases in value $80 per month.

a. Make a table for the value of the car during its first six months.
b. Write a recursive function to represent the sequence in your table.
c. Write an explicit formula to represent the sequence.
d. Does the sequence represent a function?
e. Predict the value of the car at the end of two years.
f. When will the car's value be $21000?

Case Closed - Evidence:

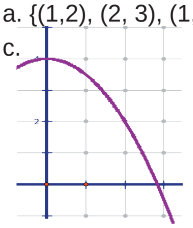
Month (***n***)	1	2	3	4	5	6
Car value (***V***)	26500	26420	26340	25260	26180	26100

b. ***V***1 = 26500   ***Vn*** = ***Vn*** -1 - 80   c. ***V*** = 26500 – 80(***n*** – 1)
d. The sequence represents a linear function because it has a constant rate of change.
e. ***V*** =26500 – 80 (24 - 1) = 24660   The value of the car will be $24660.
f. 21000 =26500 - 80(***n*** – 1) In 69.75 months, the car's value will be $21000.

3. Which of the following relations are also functions?

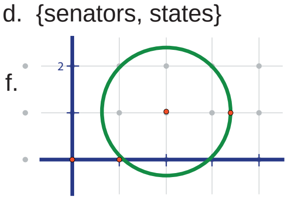
a. {(1,2), (2, 3), (1, 4), (4,1)}   b. ***y*** = 3***x*** + 22

c.

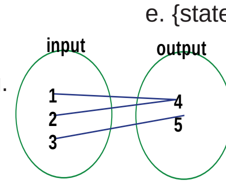


d. {senators, states}   e. {states, senators}

f.



g.



Case Closed - Evidence:

a. Not a function because 1 has 2 different outputs. b. Linear function c. Non-linear function d. Function; every senator represents one state. e. Not a function; every state has more than one senator. f. Not a function; fails vertical line test. g. Function because every input value corresponds to just one output value.

Further investigations:

Discuss relationships between varying quantities with your student. Decide if the relationship is a linear function. Here are some examples: miles driven and amount of gasoline remaining in the tank; grams of fat in one serving and calories in that serving; number of friends and amount of one shared pizza each friend gets; number of minutes a player is in a basketball ame and the number of points that player scores.

Pretend you just won $50,000 in the Lottery and you want to split most of your winnings between paying down your mortgage and saving for your student's future education. You want to put no more than $---- in the mortgage account and at least $---- in savings for education. Ask your student to graph these relationships.

Terminology:

Constant function: A relationship between two variables in which the dependent variable does not change. The graph of a constant function is a horizontal line.

Function: A relation such that each input value (x) is associated with exactly one output value (y).

Line of best fit: The line that best represents the trend established by the points in a particular scatter plot.

Linear function: A relationship between two variables in which the rate of change is constant. Graphs of linear functions are non-vertical lines.

Linear inequality: An inequality in two variables for which the graph of the solutions form a half-plane on one side of a line and may or may not also form the line itself.

Relation: A set of ordered pairs of coordinates.

Scatter plot: The graph of a collection of ordered pairs of coordinates.

Slope: The steepness of a line which may be calculated by finding the ratio of the difference between the ***y***-coordinates of two points on a line to the difference between the ***x***-coordinates of those two points.

Slope intercept form: One way to write a linear equation: ***y*** = ***mx*** + ***b*** where ***m*** is the slope and **b** is the ***y***-intercept.

Standard form: One way to write a linear equation: ***ax*** + ***by*** = ***c*** where ***a***, ***b***, and ***c*** are constants.

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Slippery Slope

Students will:

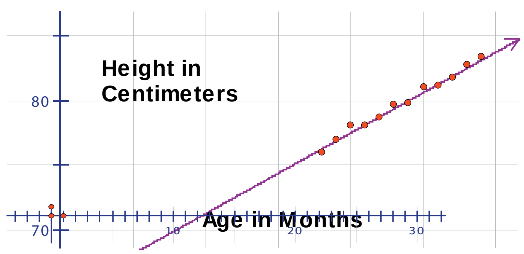
- Collect data about relationships between varying quantities, organize that data into tables, and graph the data
- Analyze tables, graphs, and equations to determine the relationship between varying quantities
- Interpret slope as how the rate of change in one variable affects the other
- Determine the meanings of slope and ***y***-intercept in a given situation
- Graph equations in slope-intercept form and standard form
- Identify functions as linear or non-linear
- Graph the open and closed, half-plane solution sets of linear inequalities
- Solve problems involving linear relationships by collecting data, graphing the data as a scatter plot, determining the line of best fit, writing its equation, and interpreting the solution of the equation in the context of the original problem.

Classroom Cases:

1. Jenny noticed that her parents had recorded her height every month in her baby book for almost 2 ½ years. A part of the measurements are shown below. Graph the data, determine the line of best fit, write its equation, and interpret the slope and ***y***-intercept.

Age in months	18	19	20	21	22	23	24	25	26	27	28	29
Height in cm.	76.1	77	78.1	78.2	78.8	79.7	79.9	81.1	81.2	81.8	82.8	83.5

Case Closed - Evidence:



I rolled a piece of spaghetti through my points until I felt I had a line that went through the data in the direction of the trend. I chose two points on the line:

(20, 77.5) and (24, 80).

Using the points, I found the slope of the line:

$$m = \frac{(80 - 77.5)}{(24 - 20)} = 0.625$$

Then I substituted in ***y*** = **mx** + ***b*** to find ***b***.

80 = 0.625 (24) + ***b***

80 = 15 + ***b***

65 = ***b***

My equation is ***y*** = 0.625x + 65. The y-intercept is 65 cm which is about Jenny's height when she was 0 months old, and the slope is 0.625 which means that as Jenny got a month older, her height increased by 0.625 cm on average. Jenny's height at birth was less than 65cm. She grew more rapidly during her first year.

2. After finishing dinner and before going to bed, Frank has 3 hours to do his homework and practice his guitar. He wants to spend more time practicing guitar than doing homework, but he has to finish a report and study for a science test, which will take at least one hour. Write inequalities to represent the conditions in Frank's situation and graph each inequality on a different pair of axes.

Case Closed - Evidence:

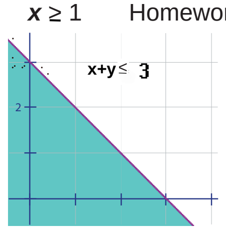
Let ***x*** = homework time

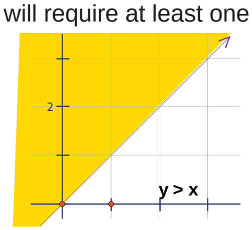
y = guitar time

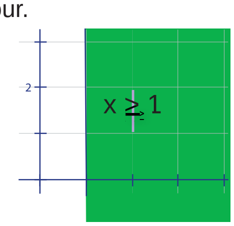
Then $x + y \le 3$ Homework time and guitar time must be less than or equal to three hours.

$y > x$ He wants to spend more time on guitar than on homework.

$x \ge 1$ Homework will require at least one hour.







Further investigations:

Challenge your student to identify congruent angles in brickwork, tiling, wallpaper and textile design, carpentry, and artwork. Discuss the math that supports each pair of congruent angles.

When you are planning home repairs or renovations, invite your student to participate. Encourage her to “do the math” that is needed to assure right angles will be formed, shelves will be parallel to the floor, fence posts will be perpendicular to the ground, and wall décor will show symmetry.

If your student is involved in a service organization such as Scouts, 4-H, Beta Club, or church youth group, suggest that the organization engage in a service project that allows members to build objects with parallel sides, such as collection bins for food or clothes or books. Consider making and selling bird houses or picture frames as a fundraiser for a special project. Members will use their understanding of congruent angles informally as they plan and complete the constructions.

Terminology:

Adjacent angles: Angles in the same plane that have a common vertex and a common side, but no common interior points.
Alternate angles: Pairs of angles formed when a transversal crosses two other lines. These angles are on opposite sides of the transversal. Alternate exterior angles are outside the other two lines. Alternate interior angles are between the other two lines.
Complementary angles: Two angles whose sum is 90°.

Congruent: Having the same size, shape, and measure. Two figures are congruent if all their corresponding measures are equal.
Corresponding Angles: Angles that have the same relative positions in geometric figures.

Linear pair: Adjacent, supplementary angles. A linear pair forms a line.

Skew lines: Two or more lines that do not lie in the same plane. Skew lines can be parallel or intersecting.

Supplementary angles: Two angles whose sum is 180°.

Transversal: A line that crosses two or more lines.

Vertical Angles: Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

Related Files:

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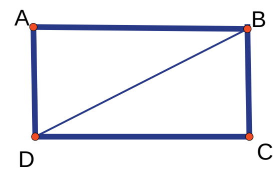
Parallel Lines and Congruence

Students will:

- Investigate and use characteristics of parallel and perpendicular lines
- Apply properties of angles and segments for parallel lines cut by one or more transversals
- Construct parallel and perpendicular lines using Euclidean tools
- Identify congruent figures using similarity ratios, reflection images, or congruent corresponding parts.

Classroom Cases:

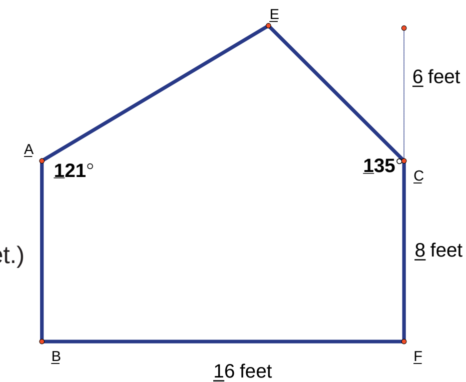
1. Dion attached a turnbuckle (BD) to the gate to help maintain the shape of the gate. Dion wants to keep the top and bottom rails parallel and the side rails parallel. How can Dion check to see if the turnbuckle is doing its job?



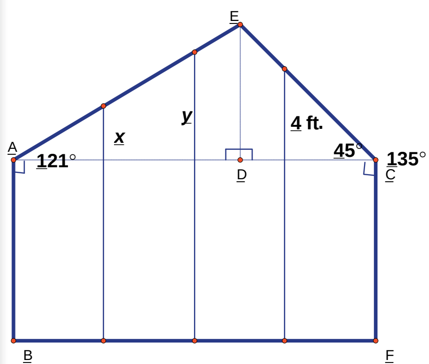
Case Closed - Evidence:

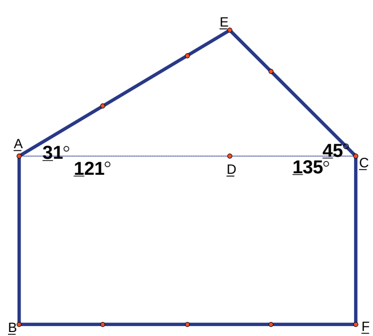
We can think of the top and bottom rails ($\overline{AB}$ and $\overline{DC}$) and the side rails ($\overline{AD}$ and $\overline{BC}$) as line segments. The turnbuckle ($\overline{BD}$) acts as a transversal. Dion needs to measure alternate interior angles ∠ABD and ∠CDB. If they have the same measures (are congruent), then the top and bottom rails are parallel. Then he needs to measure ∠ADB and ∠CBD. If they are congruent, the side rails are parallel.

2. In the summer, Casey helps remodel and restore houses. For the current project, Casey must cut paneling for the family room. One wall is sketched at right. Help Casey determine how long to cut the paneling sheets and at what angles for the wall shown.
(Paneling comes in sheets that are 4 feet by 8 feet.)



Case Closed - Evidence:





If I draw $\overline{AC}$, I will form rectangle ACFB. Four sheets of paneling will cover this area. I can form two triangles by constructing segment DE ⊥ AC through point E. △CDE is an isosceles right triangle because ∠ECD =45°. (180 -135 =45). DC ≅ DE and both have measures of 6 feet as shown in the sketch above. A 4-foot wide piece of paneling will fit 4/6 of the way from C to D. The side of the paneling must be 4/6 as long as DE because the side of the paneling and DE are parallels cut by transversal CE and the segments must be proportional. Similarly, the sides of the paneling to cover △ADE are parallel to DE and their lengths will be proportional to the their distances from A. AD = 10 feet since 16 - 6 =10. x =4/10 of 6 = 2.4 feet and y =8/10 of 6 = 4.8 feet.

In △AEC, ∠ECA = 45° (135-90=45) and ∠CAE = 31° (121-90=31). So, ∠AEC =180- (45+31) = 104°.

Book ‘em:

Euclid’s Window: The Story of Geometry from Parallel Lines to Hyperspace

by Leonard Mlodinow

Further investigations:

Encourage your student to compare payment plans from two cell phone companies. Suggest that she graph the monthly service charge and per-minute fees for each company. Based on how she might use a cell phone, which plan is best for her? Which plan would be best for someone who has no other telephone? Which plan would be best for someone who only uses a cell phone for emergencies? When would the plans have the same charge?

When your student is interested in purchasing something, ask him to represent the options mathematically. For example, with $10.00, what combinations of hamburgers and fries can he buy?

With your student, look at graphs that have more than one line. Ask her what is happening when the lines cross each other? What would it mean if they never cross? When would one line be the better choice than the other and why?

Terminology:

Constraint: A restriction placed on variables in a problem situation. Inequalities can represent constraints.

Feasible region: The area on a graph consisting of all the points that satisfy the constraints.

Graphing method: A technique for solving a system of equations that involves reading the coordinates of the point of intersection of the graphs of the equations in the system

Combination method: A technique for solving a system of equations that involves combining two equations in order to eliminate one of the variables and solving for the remaining variable.

Linear inequality: A mathematical statement which expresses a relationship between two unequal linear expressions. The graph of a linear inequality is a half-plane on one side of a related line.

Substitution Method: A technique for solving a system of equations that involves replacing one variable with an equivalent expression and solving for the remaining variable.

System of linear equations: Two or more equations that together define a relationship between variables usually in a problem situation. A system of equations can have no solution, one solution, or many solutions.

System of linear inequalities: Two or more inequalities that together define a relationship between variables, usually in a problem situation.

Related Files:

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Systems of Equations and Inequalities

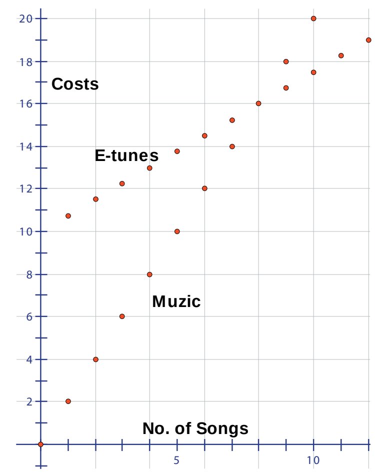
Students will:

- Solve systems of equations geometrically and algebraically
- Determine the number of solutions a system of equations will have before attempting to solve the system
- Translate a problem situation into a system of equations or inequalities
- Interpret the solution to a system of equations in terms of the original problem
- Determine the solution to a system of linear inequalities by graphing

Classroom Cases:

1. E-tunes and Muzic provide music download services. E-tunes charges a $10 annual fee and $.75 for each song downloaded. Muzic has no annual fee, but charges $2.00 per song. Which service would you prefer and why?

No. of songs	E-Tunes	Muzic
1	$10.75	$2.00
2	$11.50	$4.00
3	$12.25	$6.00
4	$13.00	$8.00
5	$13.75	$10.00
6	$14.50	$12.00
7	$15.25	$14.00
8	$16.00	$16.00
9	$16.75	$18.00
10	$17.50	$20.00



Case Closed - Evidence:

I would prefer E-tunes because I plan to download at least one song a month and E-tunes is cheaper for 9 or more songs. Both services cost the same for 8 songs.

2. Six hundred tickets were sold for the Spring Concert, and the total income from ticket sales was $2716.50. If adult tickets cost $5.25 and student tickets cost $ 3.75, how many of each type of ticket were sold?

Case Closed - Evidence:

I will let $\boldsymbol{a}$ = number of adult tickets and $\boldsymbol{s}$ = number of student tickets

Number of tickets: $\boldsymbol{a} + \boldsymbol{s} = 600$

Value of tickets: $5.25\boldsymbol{a} + 3.75\boldsymbol{s} = 2716.50$

From the first equation, $\boldsymbol{s} = 600 - a$.

Substituting, $5.25\boldsymbol{a} + 3.75(600 - \boldsymbol{a}) = 2716.50$

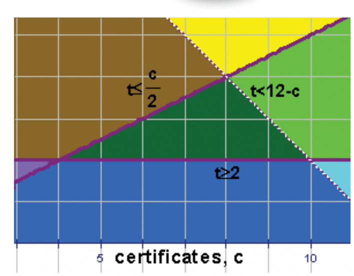
$5.25\boldsymbol{a} + 2250 - 3.75\boldsymbol{a} = 2716.50$

$1.5\boldsymbol{a} = 466.50$

$\boldsymbol{a} = 311$

$\boldsymbol{s} = 600 - \boldsymbol{a} = 600 - 311 = 289$

3. Coach Woods is buying awards for the end of season banquet. Based on past experience, she knows she will need at least twice as many certificates as trophies, at least 2 trophies, and fewer than 12 awards. If certificates cost $3 and trophies cost $15, what is the most Ms. Woods could spend on awards and what is the least she could spend?



Case Closed - Evidence:

I graphed the situation constraints as shown at right. There are infinitely many values in the feasible region (dark green triangle), but only whole numbers make sense here. The fewest awards Ms Woods can buy is 6 certificates and 2 trophies; this will cost $48. The most awards the coach can buy is 11; the most expensive combination is 2 trophies and 9 certificates for $57