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CSI: MATHEMATICS

Curriculum Support Information

A mathematics resource for parents, teachers, and students

GEORGIA DEPARTMENT OF EDUCATION
Kathy Cox, State Superintendent of Schools

Further investigations:

With your student, find graphs and summary statistics in newspapers and magazines. Discuss the messages delivered by these displays and analyses of data.

Ask your student to record for 10 days the amount of minutes he spends each day on two non-school activities (such as watching TV and listening to music). Together, make a double box plot and compare how your student is spending his leisure time.

Use the data collected on leisure time activities to make a scatter plot that will answer the question, “Does time spent on activity 1 depend on time spent on activity 2?”

Terminology:

Box plot: A diagram that summarizes data using the median, upper and lower quartiles, and extreme values.

Census: Collection of data from every member of a population.

Five number summary: Minimum, lower quartile, median, upper quartile, maximum of a data set.

Interquartile range: The difference between the first and third quartiles.

Measures of center: Numerical values used to describe the clustering of data in a set. Mean, median, and mode are common measures of center.

Measures of variation: Numerical values used to describe the spread or dispersion of data in a set. Range and interquartile range are common measures.

Outlier: A value that is very far away from most of the values in a data set.

Parameter: A measured characteristic of a population.

Quartiles: Numbers that divide data into quarters when data in a set are in order.

Sample: A selected part of a population.

Scatter plot: A graph of a set of ordered pairs.

Statistic: a measured characteristic of a sample.

Related Files:

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Dealing with Data

Students will:

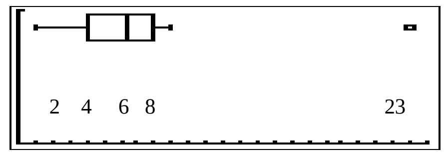
- Form questions answered by data and collect data from a sample and from a population to answer the questions
- Display data in appropriate graphs, including box plots and scatter plots
- Analyze data using measures of center and measures of variation
- Describe how a change in one variable affects another variable
- Discuss how sample size affects sample statistics
- Compare data sets using graphs and statistics

Classroom Cases:

1. The following list shows the ages of cars (in years) in the faculty/staff parking lot: 4, 9, 9, 8, 7, 1, 5, 4, 4, 4, 7, 6, 7, 23. Find the 5-number summary and construct a box plot. Identify at least one interesting characteristic.

Case Closed - Evidence:

5-number summary	
Minimum	1
Lower quartile	4
Median	6.5
Upper quartile	8
Maximum	23

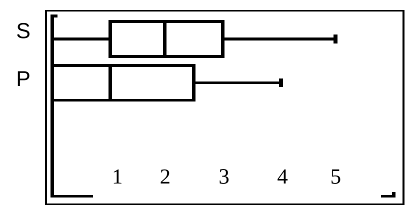


75% of the cars are 8 years or less in age. One car, probably a classic, is an outlier at 23 years old.

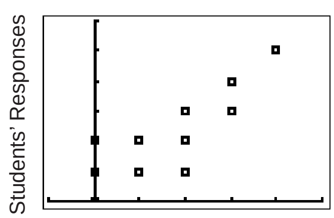
2. Who drinks more soft drinks: 7th graders or their parents? Conduct a survey to gather data to answer this question, display the data appropriately, and analyze it to answer the question.

Case Closed - Evidence:

I interviewed 24 7th graders and one of their parents. I found that 7th graders drank more soft drinks a day than their parents did. The mean number of sodas for 7th graders was 2 compared to 1.46 sodas per day for the parents. As the box plots at right show, the highest number of sodas for students was 5 but the highest for parents was only 4. The lowest for both groups was 0, but 29 % of parents responded, “0” (that is they, drank no soft drinks). Also for parents, the median was only 1, meaning that 50% of my sample drank 1 or fewer soft drinks a day. For students, the median was 2, so 50% of the students I surveyed drank 2 or fewer sodas a day and 50% drank 2 or more.



Cans of Soda



Parents’ Responses

In the graph at left, I plotted parents’ responses on the horizontal axis and students’ responses on the vertical axis because I wanted to see if the number of sodas students drank depended on the number their parents drank.
Since the points sort of make a line, I think that the more sodas parents drink, the more sodas their children drink. There seems to be a positive association.

3. Abby, Ben, and Carlos conducted the soda survey described above. Abby asked 12 people on the bus on the way home. Ben surveyed 35 people at the mall. And Carlos called 65 people on the telephone. Discuss how different sample sizes and survey techniques could affect their results.

Case Closed - Evidence:

Abby only surveyed 12 people. Her sample is small and it comes from a limited group. It may not accurately reflect the true characteristics of the population. Ben surveyed more people, but only those who could find rides to the mall. Ben’s sample may not represent people who shop online. Carlos surveyed a fairly large sample and should therefore be able to avoid an impact from unusual behaviors. Most people in the U.S. have telephones, so his sample would allow all members of the population to be represented. If he selected phone numbers randomly, there should be no sampling bias. I think Carlos’ sample will be an accurate representation of the population of 7th graders and their parents

Further investigations:

Using data from your student's baby book, work with her to make a scatter plot of her height over time. When did she grow the fastest? What would the plot look like if there had been no growth in a particular time period? Could the plot decrease? What would that mean?

Look for tables and scatter plots in newspapers and magazines. With your student, identify the variables. Which one is independent? How do you know? Discuss how the variables are related. Talk about what the graph shows. Could a table provide the same information?

Identify relationships between varying quantities in everyday experiences and discuss which quantity depends on the other. For example, consider temperature and time of day, inches of rainfall and the height of grass, price of movies and number of people attending. Are the relationships likely to be linear (have a constant rate of change)? Are they increasing (as one increases, the other increases)?

Using scores from football games, challenge your student to write algebraic expressions that could produce those scores. For a score of 28, a team could have made 4***t*** + 4***p*** (4 touchdowns and 4 points after) or 2***t*** + 2***p*** + 4***f*** + 1***s*** where ***f*** represents field goal and ***s*** stands for safety.

Terminology:

Algebraic expression: Mathematical phrase involving at least one variable.

Dependent variable: The quantity being measured or counted whose value is determined by the independent variable.

Equation: A mathematical sentence that shows that two expressions represent the same value.

Independent variable: The quantity being measured or counted whose value is determined by choice.

Linear equation: An equation in which all the variables have exponents of 1 and none of the variables is multiplied by other variables.

Rational number: A number that can be written as ***a/b*** where ***a*** and ***b*** are integers, but ***b*** does not equal 0.

Real numbers: All the rational and the irrational numbers.

Variable: A symbol (often a letter) that represents a number.

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Patterns and Relationships

Students will:

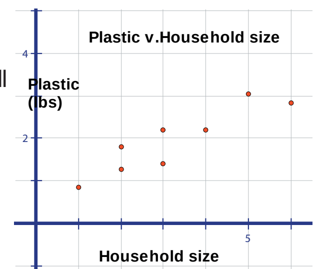
- Collect, organize, and graph data that relates two variables
- Analyze graphs and tables to determine relationships between quantities
- Represent relationships with descriptions, tables, graphs, and equations
- Translate verbal phrases to algebraic expressions and simplify using properties of Real numbers
- Use addition and multiplication properties of equality to solve linear equations
- Solve problems by defining a variable, writing and solving an equation, and interpreting the solution in the context of the problem

Classroom Cases:

1. The table below lists weights (in pounds) of plastic discarded by a sample of households along with the size of the households. Make a scatter plot and discuss the relationship between the variables.

Plastic (lb)	1.27	1.41	2.19	2.83	2.19	1.81	0.85	3.05
Household size	2	3	3	6	4	2	1	5

Case Closed - Evidence:



I think the amount of plastic discarded depends on the number of people in the household. So, pounds of plastic will be the dependent variable and number in household will be the independent variable. Household size will be my horizontal axis and plastic (lbs.) will be the vertical axis. Since my data values are small, I will scale my axes in increments of 1. Because household sizes are measured in Whole numbers (we don't have 2.3 households), the data is discrete and I will not connect the points.

Since the points go up from left to right as I look at the graph, the relationship between household size and pounds of plastic discarded is increasing. More people throw away more plastic.
The points seem to form a line. So the relationship is linear. That means the amount of plastic thrown away increases by about the same amount each time the household size increases by one person.
The relationship between household sizes and pounds of plastic discarded is linear and increasing.

2. Esta has four cousins. Let ***a*** represent Esta's age. Represent her cousins' ages with algebraic expressions in terms of Esta's age. If Esta is 10 years old, how old are her cousins?

A. Paul is 4 years more than twice Esta's age.
B. Sara is 2 years younger than Paul.
C. Mona is half as old as Sara.
D. Luis's age is 100 less than the square of Mona's age.

Case Closed - Evidence:

Cousins' Ages	
Paul: 2***a*** + 4	2•10 + 4 = 20 + 4 = 24
Sara: Paul -2 = (2***a*** + 4) - 2 = 2***a*** + 4 -2 =2***a*** + 2	2•10 + 2 = 22
Mona: 1/2 (Sara) =1/2 (2***a*** + 2) = ***a*** + 1	10 + 1 = 11
Luis: $(\text{Mona})^2$ -100 = $(a + 1)^2$ -100	$(10 + 1)^2 - 100 = 11^2 - 100 = 121 - 100 = 21$

3. Kiki wants to buy an MP3 player. She has saved $22 so far. She makes $20 per week by tutoring younger neighbors. She saves $14 of her earnings each week. If the MP3 player costs $158.00, when will Kiki be able to make her purchase? (Show how you know.)

Case Closed - Evidence:

$ she has + $ she saves per week • number of weeks > cost of MP3 player

I know values for everything in the sentence above except number of weeks.
So let ***w*** = number of weeks. Then I substitute to get:

$22 + 14 \cdot w \geq 158$ — I begin solving by subtracting

$-22 \qquad -22$

$14w \geq 136$ — Last, I divide by 14.

$w \geq 9.71 \approx 10$

Kiki will have enough money to buy the MP3 player in 10 weeks

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Further investigations:
Help your student record the daily changes in two or three stocks.

Play board and card games with positive and negative numbers such as Monopoly, Integer War, and Spades.

When watching football games, guide your student in recording gains and losses of yardage.

Discuss real life applications of positive and negative numbers such as temperature, checkbook, and height above or below sea level.

Terminology:
Absolute value: The distance between a number and zero on the number line.
|-8| =|8| = 8

Additive inverse: The value added to a number to give a sum of 0. A number and its additive inverse form a zero pair.

Distributive property: The sum of two addends multiplied by a number will be the sum of the product of each addend and the number.

$a(b+c) = ab + ac$

Integers: The set of whole numbers and their opposites. {…, -3, -2, -1, 0, 1, 2, 3, …}

Multiplicative inverse: The value multiplied by a number to give a product of 1. Reciprocals are multiplicative inverses. Multiplicative inverses always have the same sign.

Natural numbers: The set of numbers {1, 2, 3, 4, …}

Negative numbers: The set of numbers less than 0.

Opposite numbers: Two different numbers that have the same absolute value -5 and 5

Positive numbers: The set of numbers greater than 0.

Quadrant: One of the four sections into which the coordinate plane is divided by the ***x***– and ***y***– axes.

Rational numbers: The set of numbers that can be written ***a***/***b*** where ***a*** and ***b*** are integers and ***b***≠0.

Whole numbers: The set of numbers {0, 1, 2, 3, 4, …}

Zero pair: A number and its opposite. The sum of a zero pair is 0. -7 + 7 = 0

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Rational Reasoning

Students will:
Seventh Grade 3 of 7

- Compare and order positive and negative numbers, plot them on the number line, plot them in the coordinate plane, and use absolute value to explore the relationship between a number and its additive inverse
- Investigate positive and negative numbers in real contexts and develop models and algorithms for computing with them
- Apply properties of real numbers and order of operations to simplify and evaluate algebraic expressions involving positive and negative rational numbers
- Solve problems by writing and solving equations and interpreting their solutions.

Classroom Cases:
1. Kumar was playing a game. He had 34 points, but then he lost points on three turns and found that he had only 22 points. Draw a model to represent this problem and use it to find out how many points, on average, Kumar lost on each turn. Write and solve an equation to determine the number of points lost per turn.

Case Closed - Evidence:

34 points	Lost points	Lost points	Lost points

22 points

34 - 22 = 12

Lost points	Lost points	Lost points

12 points

12 / 3 = 4 On average, 4 points were lost on each of the last 3 turns.

Let p = the points lost per turn.

Then $34 + 3\boldsymbol{p} = 22$

$-34 \quad -34$

$\frac{3\boldsymbol{p}}{3} = \frac{-12}{3}$

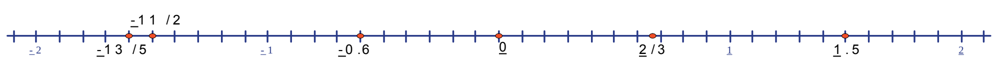
$p = -4$ Kumar lost 4 points per turn.

2. Graph these numbers on a number line and answer the questions: $\frac{2}{3}$, -0.6, $-1\frac{1}{2}$, 1.5, 0, $-1\frac{3}{5}$.

A. Which number has the greater absolute value: 1.5 or $-1\frac{3}{5}$? How do you know?

B. Which of the fractions and mixed numbers represent terminating decimals?

Case Closed - Evidence:



A. $\left|-1\frac{3}{5}\right| = 1\frac{3}{5} = 1.6$ $|1.5| = 1.5$ Since 1.6 is farther to the right on the number line than 1.5, $\left|-1\frac{3}{5}\right| > |1.5|$.

B. $-1\frac{1}{2}$ and $-1\frac{3}{5}$ Since the denominators of these mixed numbers are factors of 100 (or a power of 10), the mixed numbers represent terminating decimals: -1.5 and –1.6 respectively.

3. Evaluate the following expressions when a = -3, ***b*** = 5, and ***c*** = -4.

A. $\boldsymbol{ac} - 2\boldsymbol{b}$ B. $4\boldsymbol{a}^2 - 1$ C. $2(\boldsymbol{b} - \boldsymbol{c}) - 6\boldsymbol{a}$ D. $3\boldsymbol{b} + (7 - \boldsymbol{a})^2 - 5\boldsymbol{b}$

Case Closed - Evidence:

A. $-3 \cdot -4 - 2 \cdot 5 = 2$ B. $4 \cdot (-3)^2 - 1 = 35$ C. $2(5-(-4)) - 6 \cdot -3 = 36$

D. $3 \cdot 5 + (7 - (-3))^2 - 5 \cdot 5 = 90$

Book'em:
The Number Devil - A Mathematical Adventure by Hans Magnus Enzensberger

Flip, Slide and Turn

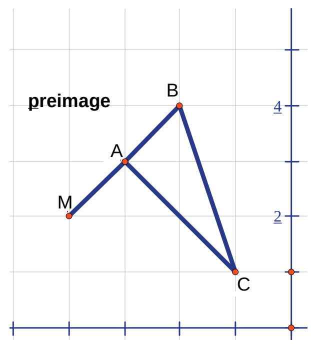
Students will:

- Analyze properties of reflections (flips), translations (slides), and rotations (turns).
- Construct reflections, translations, and rotations using coordinate geometry.
- Explore relationships of reflections, translations, and rotations using appropriate technology and manipulatives

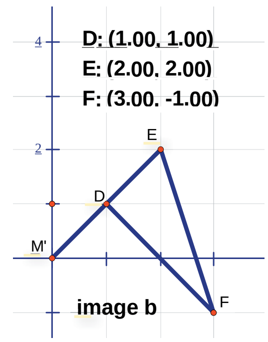
Classroom Cases:

1. a. On a grid, plot the triangle with vertices A (-3, 3), B (-2, 4), and C (-1, 1). Extend BA to M (-4, 2).
b. Translate the figure to create its image at the coordinates (***x***+4, ***y***-2).
c. Reflect the figure using the ***y***-axis as the reflecting line. Label the vertices and list their coordinates.
d. Rotate the figure clockwise 90° about the origin and label the vertices. List their coordinates.

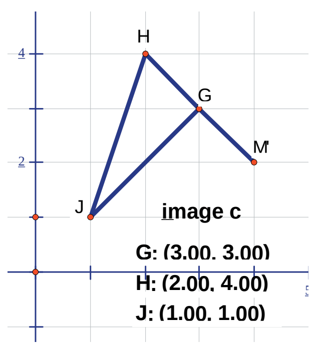
Case Closed - Evidence:



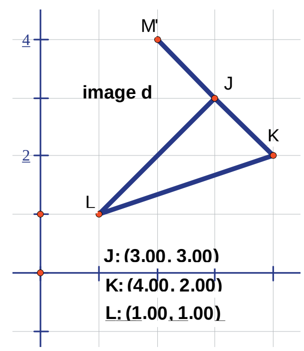
a. preimage



b. translation right 4 units, down 2 units

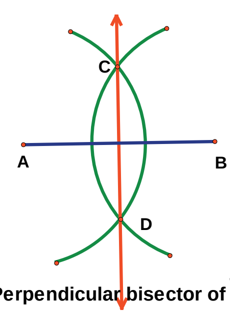


c. reflection over ***y***-axis



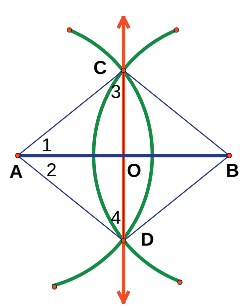
d. rotation about the origin

2. Many constructions are based on congruent triangles. Identify congruent triangles in the figure at right and tell how they are used to support the geometric construction.



Perpendicular bisector of $\overline{AB}$

Case Closed - Evidence:



ΔABC and ΔBDA are congruent and ΔACD and ΔBDC are congruent. Since the triangles are congruent, all the corresponding parts are conguent: ∠1 ≅∠2 and ∠3≅∠4. Then ∠AOC must be congruent to ∠AOD because they are the third angles in the congruent triangles Δ DOA and Δ COA. Since these angles form a line, the sum of their measures is 180°, and since they are congruent, each angle must be 90°. This means $\overline{AB} \perp \overline{CD}$. Δ COA ≅Δ COB. So AO ≅ BO. This means that CD bisected AB, that is, cut it into two equal segments.

Further investigations:

Look for patterns in fabrics, wallpaper, floor covering, architecture, etc. in which a basic shape is repeated through turns, flips, or slides. Ask your student to identify these transformations.

Ask your student to make designs that use flips, turns, or slides of a basic shape and to explain to you how he made the design.

Ask your student to create a holiday card using only a compass and straight edge. (Suggestion: Consider snowflakes; they are forms of hexagons.)

Watch while your student makes many different regular polygons with her compass and straight edge.

Terminology:

Angle of rotation: The amount of rotation about a fixed point.

Bisector: A bisector divides a segment or angle into two equal parts.

Congruent: Objects having the same size, shape and measure. ∠A ≅ ∠B denotes that ∠A is congruent to ∠B.

Corresponding Angles and Sides: Angles and sides of figures that have the same relative positions in the figures.

Line symmetry: A property of a figure that allows it to match exactly when folded in half

Point symmetry: A property of a figure that allows it to align with itself after it has been rotated about a fixed point.

Reflection: A transformation that "flips" a figure over a line of reflection.

Reflection line: A line that acts as a mirror or perpendicular bisector so that corresponding points are the same distance from the mirror.

Rotation: A transformation that "turns" a figure about a fixed point through a given angle and a given direction.

Translation: A transformation that "slides" each point of a figure the same distance in the same direction.

Transformation: The mapping, or movement, of all the points of a figure in a plane according to a common operation.

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Further investigations:

When you make copies at an office supply store, look at the enlargement/reduction feature. Ask your student to explain how it should be set to enlarge a document and how it should be set to reduce it.

Look in your kitchen for similar objects such as dishes. Ask your student to determine the scale factor relating two such objects and to determine the ratio of their areas.

Help your student measure the length and width of a large object such as a car. Ask her to determine the dimensions of a model of that object using a scale factor of 1:64.

When you and your student read the newspaper, measure the length of a head of someone in a picture. Measure the length of your own head. What scale factor could the newspaper have used? How long would your pet's head be if it were in the picture?

Find the scale located in the legend on a map of the USA. Calculate the distance from your home to Washington D.C.

With your student, watch the movie, *Honey I Shrunk the Kids*. Discuss with your student the scale factor used. How would shrinking by such a scale factor affect the surface area (or skin) of the children? How would it affect their volume or weight?

Terminology:

Congruent figures: Figures that have the same size and the same shape.

Dilation: Transformation that changes the size of a figure, but not its shape.

Proportion: An equation that states that two ratios are equal.

Ratio: Comparison of two quantities by division. A ratio may be written ***r***/***s***, ***r***:***s***, or ***r*** to ***s***.

Scale factor: The ratio of two lengths of any corresponding sides of two similar figures.

Similar figures: Figures that have the same shape, but not necessarily the same size.

Related Files:

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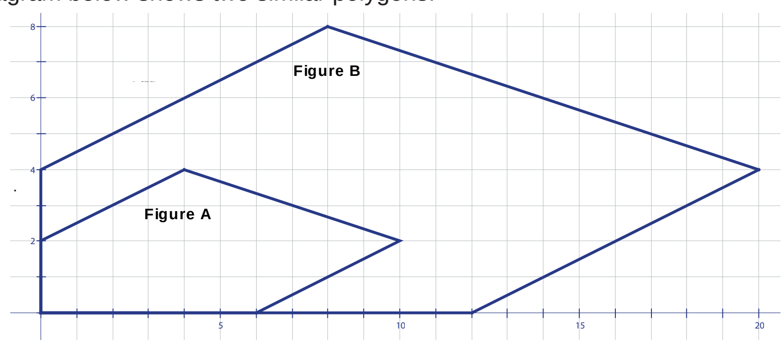
Staying in Shape

Students will:

- Enlarge or reduce geometric shapes using a given scale factor
- Given a figure in the coordinate plane, determine the coordinates resulting from a dilation
- Compare geometric figures for similarity and describe similarities by listing corresponding parts
- Describe relationships among scale factors, length ratios, and area ratios of similar geometric figures
- Use scale factors, length ratios, and area ratios to determine side lengths and areas of similar geometric figures

Classroom Cases:

1. The diagram below shows two similar polygons.



A. Write a rule for finding the coordinate of a point on Figure B from a corresponding point on Figure A.

B. Write a rule for finding the coordinate of a point on Figure A from a corresponding point on Figure B.

C. What is the scale factor from Figure A to Figure B? How are the perimeters and areas related?

Case Closed - Evidence:

A. $(x, y) \rightarrow (2x, 2y)$ B. $(x, y) \rightarrow (0.5x, 0.5y)$

C. The scale factor from Figure A to Figure B is 2 to 1. The perimeters are related by the same factor, so the perimeter of Figure B is twice as long as the perimeter of Figure A. The area of Figure B is four times the area of Figure A because the areas are related by the square of the scale factor $(2/1)^2$

2. Your principal wants to hang a banner to congratulate the basketball team on its season. The drawing for the banner is 8 inches by 15 inches. If the width of the banner will be three feet, how long should the banner be?

Case Closed - Evidence:

3 feet = 36 inches. Since the banner and the drawing will be similar, their sides must be proportional.

$$\frac{\text{Length}}{\text{Width}} = \frac{15 \text{ inches}}{8 \text{ inches}} = \frac{x}{36 \text{ inches}}$$

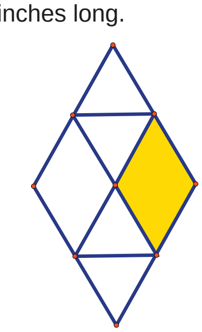
I can solve the equation by multiplying both sides of the equation by 36.

$$36 \cdot \frac{15}{8} = 36 \cdot \frac{x}{36}$$

$$67.5 = x$$

The banner should be 67.5 inches long.

3. The figure to the right is a suncatcher composed of eight congruent triangles. What is the ratio of the perimeter of the shaded area to the perimeter of the suncatcher? What is the ratio of their areas?



Case Closed - Evidence:

The perimeter of the shaded area is 4 units. The suncatcher has a perimeter of 8 units, so the ratio is 4:8 or 1:2. Since the shaded area is made up of two congruent triangles, and the suncatcher is made up of eight triangles, the ratio of their areas is 2:8 or 1:4.

Values that Vary

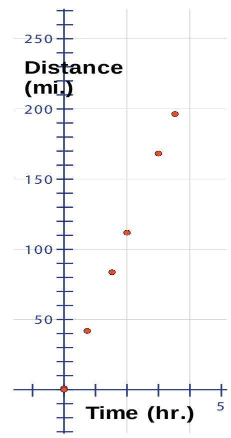
Students will:

- Collect, organize, and graph data that relates two variables
- Draw pictures and use manipulatives to demonstrate a conceptual understanding of proportion
- Solve problems using proportional reasoning
- Recognize and represent direct and inverse proportions in multiple ways
- Determine and interpret the constant of proportionality
- Explain how a change in one variable affects another

Classroom Cases:

1. Jean is traveling and keeping a record of her distance and time as shown.

Time (hrs)	0.75	1.5	2	3	3.5
Distance (miles)	42	84	112	168	196



Plot the data and describe the relationship between distance and time. If the relationship is proportional, determine the constant of proportionality. Write an equation to represent the relationship; use it to predict how far she will go in 8 hours.

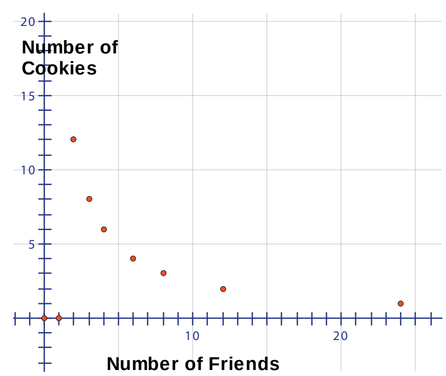
Case Closed - Evidence:
Distance and time have a direct proportional relationship. As time increases, distance increases by a multiple of 56. The constant of proportionality is distance/time or 56 miles per hour.

The relationship can be represented by $D = 56t$
where D = distance traveled and t = time traveled.
Then $D = 56 * 8 = 448$ miles traveled in 8 hours.

2. Tony just baked 24 cookies which he plans to give to his friends. He may have 1, 2, 3, or more friends. Make a table to show how many cookies each friend will get if the friends share equally. Plot the information in your table and describe your graph. If the relationship between number of friends and number of cookies is proportional, determine the constant of proportionality. Write an equation to represent the relationship and use it to predict how many cookies each person will get if Tony shares with 30 friends.

Case Closed - Evidence:

No. of friends	1	2	3	4	6	12
No. of cookies	24	12	8	6	4	2



Number of friends and number of cookies vary inversely. As friends increase, the cookies per friend decrease. The constant of proportionality is 24.
$f \cdot c = 24$ where f = no. of friends and c = no. of cookies per friend. For 30 friends, $30c = 24$ and $c = 4/5$ cookie

3. Taneisha and Marcus are having a party.

A. Taneisha is making punch. Her recipe calls for 3 cups of pineapple juice for every 5 cups of ginger ale. How many cups of each will she need to make 120 cups of punch?

B. Marcus is replenishing the punch and notices that as more guests arrive, each guest gets fewer ounces of punch. When six guests are present, one pitcher will supply each guest with 5 ounces of punch. When 10 guests are present, they empty the pitcher with 3 ounces each. How many ounces does the pitcher hold?

Case Closed - Evidence:
The relationship between pineapple juice and punch is a direct proportion as is the relationship between ginger ale and punch. There are 3 cups of juice and 5 cups of ginger ale for every 8 cups of punch.

$$\frac{\text{Juice}}{\text{Punch}} = \frac{3}{8} = \frac{j}{120} \qquad \frac{\text{ginger ale}}{\text{punch}} = \frac{5}{8} = \frac{g}{120}$$

Taneisha needs 45 cups of juice and 75 cups of ginger ale.
The relationship between guests and servings is inverse. Since 6×5oz = 30oz and 10×3oz = 30oz, 30 is the constant of proportionality and the total ounces the pitcher holds.

Further investigations:

Look for direct relationships and discuss them with your student. For example, if you earn an hourly wage, how will your earnings change as the number of hours you work changes?

Examine savings accounts with your student. How does the amount of interest earned (on a given principal at a fixed rate) change as time increases?

Review your telephone or cell phone bill with your student. How does the bill change when additional minutes are used?

Look for inverse relationships and discuss them with your student. For example, on a trip, how does travel time change when you change your rate of speed?

On a car trip, choose a distant object such as a telephone pole. Ask your student to notice how the height of the object seems to change as your car gets closer to the object. Discuss the relationship between distance away and apparent height.

Terminology:

Constant of proportionality: A value, k, that does not change; it indicates the relationship between the variables. In a direct proportion, k = the ratio of the variables. In an inverse proportion, k = the product of the variables.

Direct variation: A relationship between 2 variables in which one is the constant multiple of the other. x and y are directly proportional, if $y=kx$ where k denotes a constant of proportionality and $k \neq 0$. Direct variation is sometimes called direct proportion.

Inverse variation: A relationship between two variables in which the product is a constant. x and y are inversely proportional, if $xy=k$ where k denotes a constant. Inverse variation is sometimes called indirect variation or indirect proportion.

Book'em:

Chapter 5 in **The Man Who Counted** by Malba Tahan

Related Files:

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Further investigations:

Invite your student to help you make dinner. What two dimensional shapes can he make by slicing a cucumber, a carrot, a block or wedge of cheese, and jellied cranberry sauce?

Ask your student to identify 3-D objects in the neighborhood. Ask her how the objects could be built by stacking 2-D shapes. Then discuss what the cross-section would be if the object were sliced by a plane. For example, a chimney may be a right rectangular prism. It could be built by stacking congruent rectangles. When the chimney is sliced by the plane of the roof, the cross section is a parallelogram. Think about a church steeple, a fire hydrant, and dormers.

Play an advanced version of "I Spy." The objects spied must be described as plane figures translated or rotated through space. For example, a wastebasket might be a trapezoid with bases of 8 inches and 12 inches and height of 14 inches rotated about its midline.

Terminology:

Cross section: A plane figure obtained by slicing a solid with a plane.

Cylinder: A three-dimensional object with two parallel congruent circular bases.

Lateral faces of a pyramid: Faces that intersect at the vertex.

Lateral faces of prism: Faces that are not the bases of the solid.

Oblique figure: Prisms and cylinders with bases that are not aligned one directly above the other. Pyramids and cones with apex that are not aligned above the center of the base.

Polyhedron: A collection of polygons joined at their edges. Each of these polygons is called a "face."

Prism: A polyhedron with two parallel and congruent faces and all other faces that are parallelograms.

Right figure: Prisms and cylinders with bases that are aligned one directly above the other. Pyramids and cones in which the apex lies on the perpendicular line that passes through the center of the base.

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Slices and Shadows

Students will:

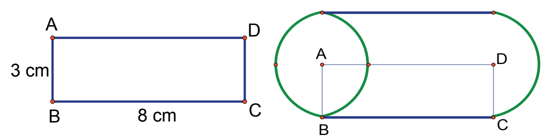
- Create three-dimensional (3-D) objects by translating (sliding) and/or rotating (turning) two-dimensional plane figures
- Explore two-dimensional (2-D) cross sections of cylinders, cones, pyramids, and prisms

Classroom Cases:

1. Construct a rectangle that is not a square from cardboard. Label the vertices in order A, B, C, D. Cut out the rectangle and rotate it 360° about side AD.

a. What shape have you formed?
b. What is the volume of your shape?
c. What is the surface area?

Case Closed - Evidence:



a. The shape I form is a cylinder. It has 2 circular bases. b. The volume tells how much space the cylinder occupies. It can be calculated by multiplying the area of one of the bases times the height.

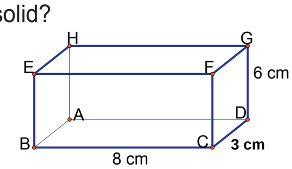
$V = \pi r^2 \cdot h \approx 3.14 \cdot 3^2 \cdot 8 = 226.08\ cm^3$.

c. The surface area tells how many square centimeters it would take to cover the cylinder. The ends of my cylinder are open so its surface is just the side or lateral area. The net for a cylinder is a rectangle with dimensions: length = circumference of the cylinder's base and width = height of the cylinder. So, the lateral area = $2\pi r \cdot h \approx 2 \cdot 3.14 \cdot 3 \cdot 8 = 150.72\ cm^2$.

2. Lay the rectangle from case 1 on a flat surface and translate it vertically or diagonally, but not horizontally.

a. What shape have you formed?
b. What is the volume of your shape?
c. What is the surface area?
d. Are the volume and surface area in this example the same as they were in case 1? Please explain.
e. How would translating in a different direction change the solid?

Case Closed - Evidence:



a. I formed a right rectangular prism. It has six rectangular faces. Opposite pairs of faces are parallel and congruent rectangles.
b. I translated the original rectangle 6 cm. My prism occupies lwh = $8 \cdot 3 \cdot 6 = 144\ cm^3$.
c. I will have to cover 2 faces that are 3 • 8 cm and 2 faces that are 3•6 cm and 2 faces that are 6 •8 cm. SA = $2 \cdot 3 \cdot 8 + 2 \cdot 3 \cdot 6 + 2 \cdot 6 \cdot 8 = 48 + 36 + 96 = 180cm^2$.
d. The volumes and surface areas are not the same. Although both solids started with the same rectangle, the transformations have created different shapes and these shapes have different dimensions that lead to different volumes and surface areas.
e. If I translated a rectangle through space perpendicular to the plane containing the original rectangle, I would get a right rectangular prism. If I translated the rectangle in a direction that is not perpendicular to the original plane, I would get an oblique rectangular prism. Its bases are rectangles and its lateral faces are non-rectangular parallelograms.

3. Make a cone from modeling clay. Use dental floss to make slices.
a. List the 2-D shapes you can make and some that you cannot make with single slices.
b. Make a slice parallel to the base halfway between the base and the vertex. Compare the top shape with your original cone.

Case Closed - Evidence:

a. From a cone, I can make circles and ellipses. I cannot make any polygons because polygons have straight sides and cones are curved.
b. My original cone and the top half of my slice are similar cones. The cross section is a circle with a radius 1/2 the length of the radius of the base of the original cone.